

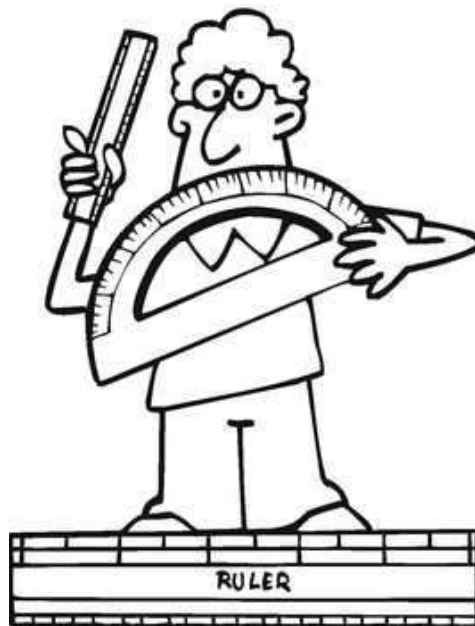
Mathematics Examination – 563-306

Secondary Cycle Two Year One

June 2010

Mathematics (Secondary 3)

Competency 2 and Competency 3
Task Booklet



**Student
Booklet**

Name : _____

Group : _____

Time: 3 hours



The following criteria will be used to evaluate your level of competency development in the different tasks presented in this booklet.

Evaluation Criteria Competency 2: <i>Uses Mathematical Reasoning</i>
Cr1 – Formulation of a conjecture appropriate to the situation
Cr2 – Correct application of the concepts and processes appropriate to the situation
Cr3 – Proper implementation of mathematical reasoning suited to the situation
Cr4 – Proper organization of the steps in a proof suited to the situation
Cr5 – Correct justification of the steps in a proof suited to the situation

Evaluation Criteria Competency 3: <i>Communicates By Using Mathematical Language</i>
Cr1 – Correct translation of a mathematical concept or process into another register of semiotic representation
Cr2 – Correct interpretation of a mathematical message involving at least two registers of semiotic representation
Cr3 – Production of a message appropriate to the communication context
Cr4 – Production of a message in keeping with the terminology, rules and conventions of mathematics



Instructions

1. Provide all the required information in the spaces in this booklet.
2. There are 9 questions in this booklet. For each question, you must demonstrate your reasoning to justify your answer. The steps in your procedure must be organized and clearly presented.
3. You are permitted to use graph paper, a ruler, a compass, a set square, a protractor and a calculator.
4. You may refer to the memory aid you prepared on your own before the examination. The memory aid consists of one letter-sized sheet of paper ($8\frac{1}{2} \times 11$). Both sides of the sheet may be used. Any mechanical reproduction of this memory aid is forbidden. All other reference materials are forbidden.

Note: Figures are not necessarily drawn to scale.



1. GOING TO SCHOOL

One morning, Tina and her brothers, Michael and Manoli, left the house to go to school. Tina was ready first, so she walked to school at a relaxed speed. Michael was ready second, so he jogged to school to avoid being late. Manoli was ready last. In order to get to school on time, he decided to ride his bike. All three siblings travelled at constant, but different, speeds.



The table below shows their speeds (km/hour) and the duration of their trip (hours).

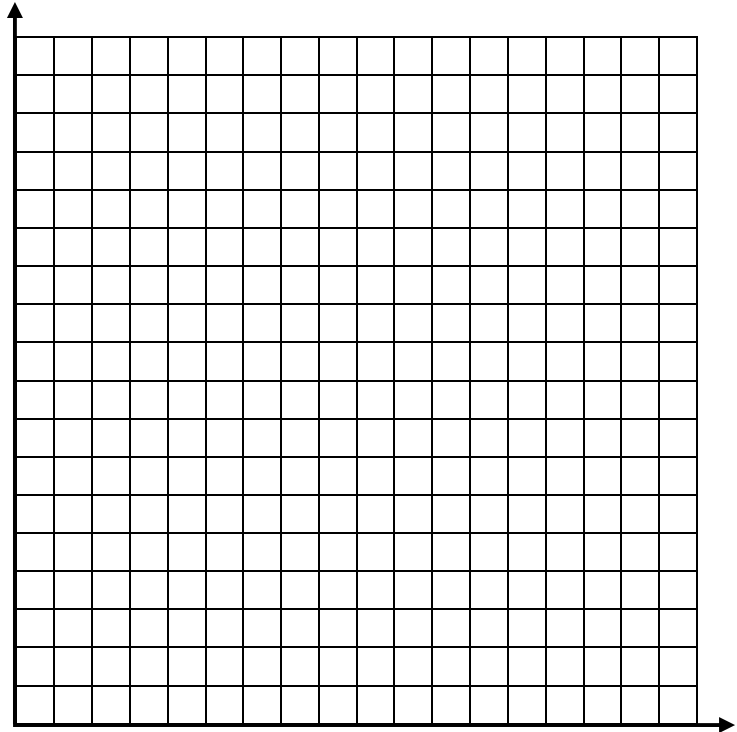
Sibling	Speed (km/hour)	Duration of Trip (hours)
Tina	5 km/hr	0.6 hours
Michael	10 km/hr	0.3 hours
Manoli	25 km/hr	0.12 hours

The next morning, their father drove them to school.

At what speed must the father drive to get to school in exactly five minutes?



Show or explain how you found your answer.
 (use graph only if needed)



The father must drive at a speed of _____ km/h to get to school in exactly five minutes.

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	40	32	24	16	8
	Cr4: Proper organization of steps Cr5: Uses sound arguments to justify the answer	20	16	12	8	4



2. COMMON FACTOR

Consider the following algebraic expressions:

Algebraic expressions
$16x^3y^4 + 3x^3y^4 - 4x^3y^4$
$\sqrt{25x^8y^6}$
$(4x^{-3}y^2)(5x^{10}y)$
$\frac{20x^{-3}y^4}{4x^{-8}y^2}$

The greatest common factor of the simplified expressions provided in the table above, can be evaluated when:

- x is 4
- y is 3

What is the numerical value of the greatest common factor given the conditions that were specified?



Show or explain how you found your answer.

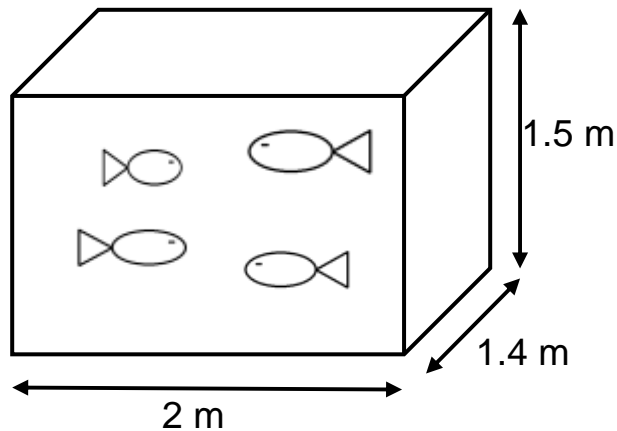
Algebraic expressions	Simplified monomials
$16x^3y^4 + 3x^3y^4 - 4x^3y^4$	
$\sqrt{25x^8y^6}$	
$(4x^{-3}y^2)(5x^{10}y)$	
$\frac{20x^{-3}y^4}{4x^{-8}y^2}$	

The numerical value of the greatest common factor is _____.

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	40	32	24	16	8
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3. MYSTIC AQUARIUM

The Mystic Aquarium is in the process of changing one of their exhibits. They want to replace one of their fish tanks with one of a different shape. Currently, the fish are in an aquarium that is in the shape of a rectangular-based right prism whose measurements are provided in the illustration below.



After doing some research, the Mystic Aquarium decides that they want an aquarium in the shape of a circular-based right cylinder with a height that is 2.6 times the radius of the base. They also want the volume of the new aquarium to be equivalent to the old one. To fulfill these requirements, they have to get the aquarium custom-made.

What are the measures of the diameter and the height of the new fish tank?



Show or explain how you found your answer.

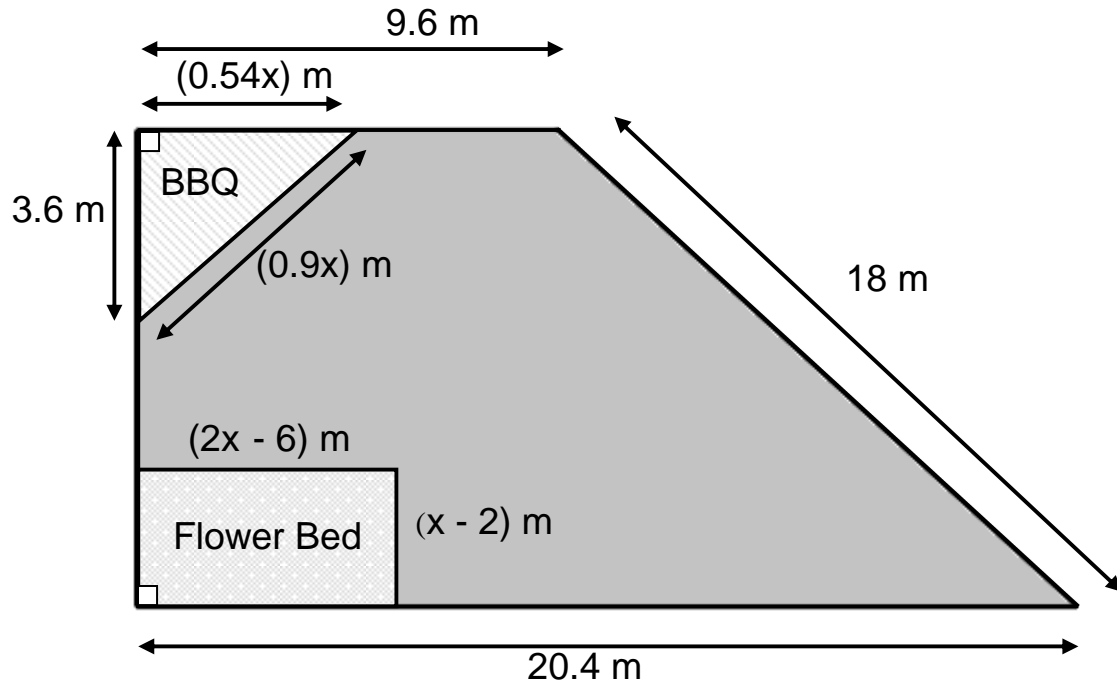
The cylindrical aquarium must
 have a diameter of _____ m
 and a height of _____ m.

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	40	32	24	16	8
	Cr4: Proper organization of steps Cr5: Uses sound arguments to justify the answer	20	16	12	8	4



4. LAWN CARE

David wants to fertilize the grass of his backyard with a new ecological brand of fertilizer that costs $\$0.80/\text{m}^2$. His backyard is in the shape of a right trapezoid as shown on the illustration below (diagram is not drawn to scale). He will not need to fertilize the parts of the backyard where the BBQ and the flower bed are located.



How much will it cost David to fertilize his backyard?



Show or explain how you found your answer.

It will cost David \$ _____ to fertilize his backyard.

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	40	32	24	16	8
	Cr4: Proper organization of steps Cr5: Uses sound arguments to justify the answer	20	16	12	8	4

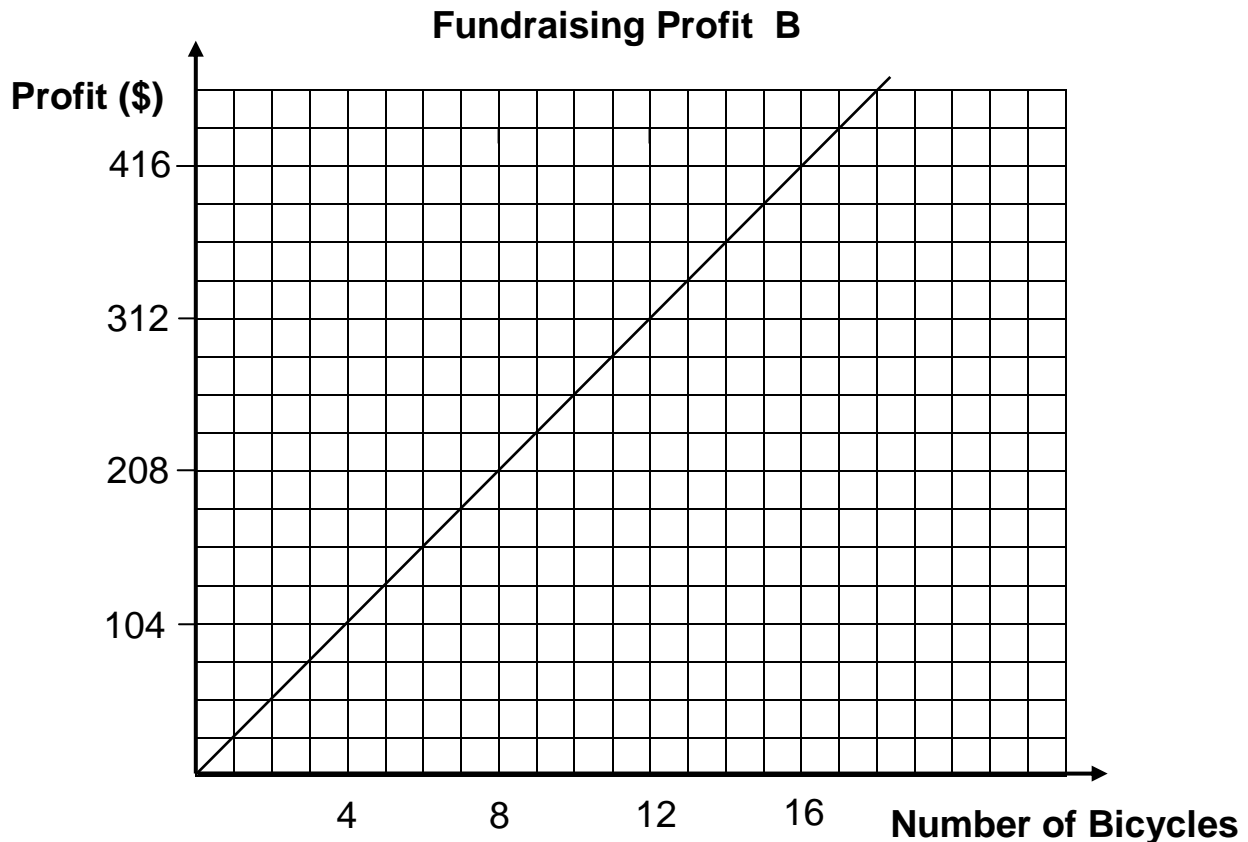


5. FUNDRAISING

In a high school, two groups of secondary students decide to raise money to help buy mosquito nets to send to Africa.

Group A invests money to buy pieces of jewellery which they will resell for profit. Each piece will be sold for the same price. They know that if they sell 85 pieces, they will make a profit of \$480. If they sell 120 pieces, they will make a profit of \$760. At the end of the campaign, they sold 90 pieces of jewellery.

Group B wants to collect used bicycles and sell them. The following graph shows the profit according to the number of bicycles sold.



Considering that Group B earned a higher profit than Group A, what was the minimum number of bicycles sold?



Show or explain how you found your answer.

The minimum number of bicycles sold by Group B is

_____.

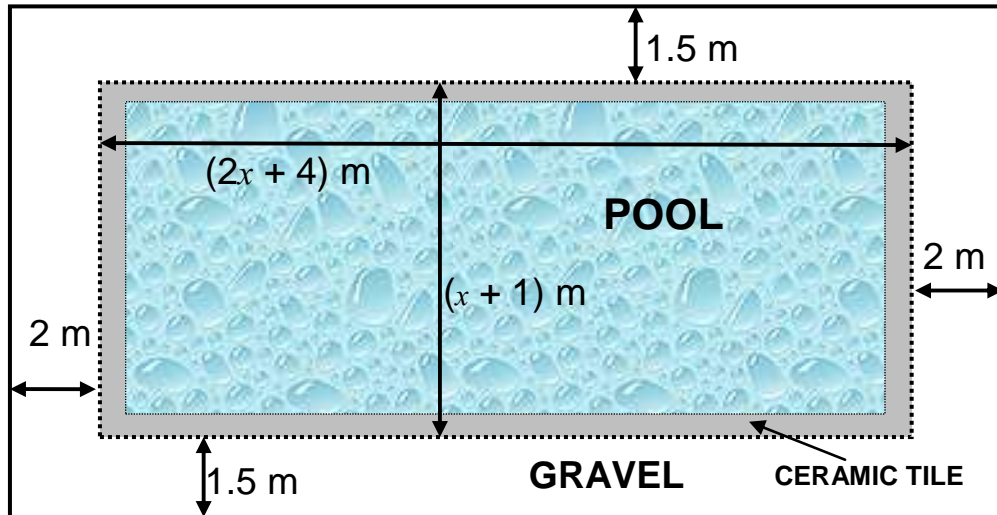
Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
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6. MAKING THE POOL SAFE

Raphael has a beautiful pool, bordered by ceramic tile, in his backyard. He wants to put a path made of gravel around the ceramic tile as shown in the picture below. The area of the gravel path will be 48 m^2 .

As a safety precaution Raphael would like to put a fence around the ceramic tile. The fence costs $\$45/\text{m}$ (taxes included).



Legend:

Fence around the ceramic tile:
.....

Border around the gravel path:

How much will Raphael have to pay to put a fence around his ceramic tile?



Show or explain how you found your answer.

It will cost Raphael \$ _____ to put a fence around his tile.

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	40	32	24	16	8
	Cr4: Proper organization of steps Cr5: Uses sound arguments to justify the answer	20	16	12	8	4



7. NEW YORK CITY



Luka is planning a trip to New York City. In order to find the best price for his trip, he researches different options offered online. In doing his research, he finds two different packages. The following is the information that he found:

Option 1: Travel Ticks

Transportation (round trip): \$250

Hotel:

Number of nights	Total cost of hotel
2	250
4	500
6	750

Option 2: Dream Tours

Transportation (round trip): \$150

Hotel: membership fee of \$25 and \$150 per night

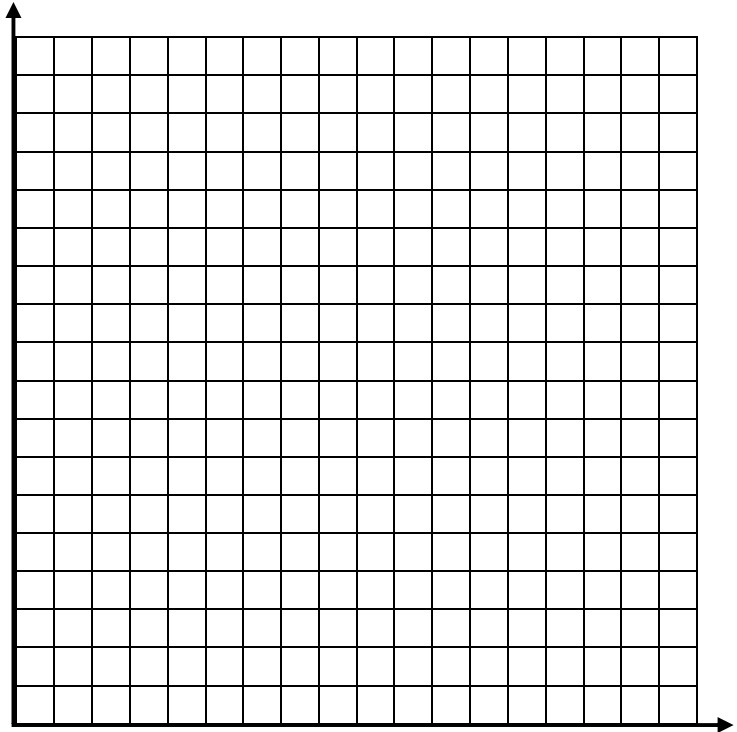
Luka is not sure how many days he will be staying in New York City.

Prepare a summary that outlines which option is the least expensive, depending on the number of nights that he will be staying in New York City.



Show your work and write your summary here.

(use graph only if needed)



Communicates by using mathematical language						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr2: Correct interpretation of a mathematical message involving at least two modes of representation	40	32	24	16	8
	Cr1: Correct translation of a mathematical concept or process into another register mode of representation	20	16	12	8	4
	Cr3: Production of an appropriate message	10	8	6	4	2
	Cr4: Using appropriate terminology & respecting the rules and conventions of mathematics	10	8	6	4	2



8. CHARITY REPORT

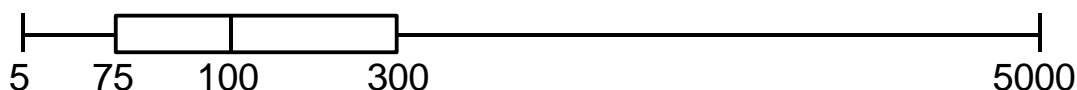


Jenny, the Director of the “*Soleil de Vie*” charity, has asked you to help prepare a report for the Board of Directors summarizing the donations they received in 2008 and 2009.

The following table gives the number of donations made in 2008:

Amount of donation (\$)	Frequency	Amount of donation (\$)	Frequency
[0, 250[5249	[1500, 1750[0
[250, 500[2500	[1750, 2000[50
[500, 750[300	[2000, 2250[0
[750, 1000[500	[2250, 2500[0
[1000, 1250[0	[2500, 2750[0
[1250, 1500[400	[2750, 3000[1

In 2009, they received 10 000 donations. The box-and-whisker plot below summarizes the data according to the amount of each donation (in dollars).



The Director would like to make the following statements to the Board of Directors comparing the donations from 2008 and 2009.

“The mean donation in 2008 was \$300, but in 2009, the mean donation was \$100.”

“In 2009, we had more people who donated between \$300 and \$5000 than the number of people who donated between \$5 and \$75.”

“For both years, however, our maximum donation was no more than \$3000.”

Explain why these statements are incorrect and rewrite them so that Jenny can present them correctly.



Show your work:

Statement 1: _____

Statement 2: _____

Statement 3: _____

Communicates by using mathematical language						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr2: Correct interpretation of a mathematical message involving at least two modes of representation	40	32	24	16	8
	Cr1: Correct translation of a mathematical concept or process into another register mode of representation	20	16	12	8	4
	Cr3: Production of an appropriate message	10	8	6	4	2
	Cr4: Using appropriate terminology & respecting the rules and conventions of mathematics	10	8	6	4	2

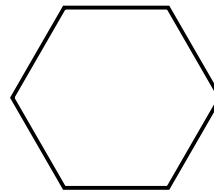
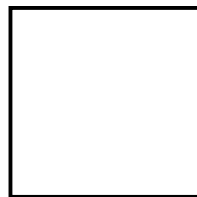
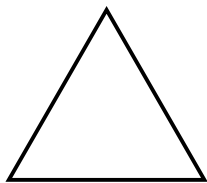


9. THE AREA OF REGULAR POLYGONS

In mathematics, a conjecture is a mathematical statement that you don't know for sure is true, but you think it might be. You can't really prove it, but you can give some examples where it looks like it's true.

You can make a conjecture by investigating mathematical relationships in numbers or in geometric shapes, and looking for some kind of pattern.

Consider a group of regular polygons whose perimeters are equal.



Make a conjecture comparing the number of sides of a regular polygon and its area.



Show or explain how you decided on your conjecture.

Conjecture:

Uses mathematical reasoning						
Observable indicators corresponding to level		A	B	C	D	E
Evaluation Criteria	Cr3: Efficient use of mathematical reasoning	40	32	24	16	8
	Cr2: Correct application of the concepts and processes	20	16	12	8	4
	Cr4: Proper organization of steps	10	8	6	4	2
	Cr5: Uses sound arguments to justify the answer					
	Cr1: Formulation of an appropriate conjecture	20	16	12	8	4

