PART 2
Marking the Tasks Focusing on the Competencies:
Uses Mathematical Reasoning
and
Communicates by Using Mathematical Language

8. Marking Key –Applications

1. GOING TO SCHOOL

Description of the task:	**Theme:** Arithmetic & Algebra
Use a rational function to determine the speed needed to get to school in five (5) minutes.	**Concepts and processes:** • Rational function of the form $y = \frac{k}{x}$ • Representing and interpreting the inverse function • Observing patterns • Finding the rules

A. Example of an appropriate solution

Rule of the function representing the duration of the trip according to the speed:

x: Speed (km/h)
y: Duration of Trip (hours)

The product of the variables is constant; therefore it is a rational function of the form $y = \frac{k}{x}$

$0.6 = \frac{k}{5}$

$k = 3$

The distance from the house to the school is 3 km.

The rule of the function is therefore: $y = \frac{3}{x}$

Check if this rule is valid for the other siblings' values:

Michael ($x = 10$):

$y = \frac{3}{10}$

$y = 0.3$ hours

Manoli ($x = 25$):

$y = \frac{3}{25}$

$y = 0.12$ hours

Speed for a 5 minute journey:

$5 \text{ minutes } = \frac{1}{12} \text{ hour}$

$y = \frac{3}{\frac{1}{12}}$

$x = 36 \text{ km / hour}$

Conclusion: The father must drive at a speed of 36 km/hour to get to school in 5 minutes.

2. COMMON FACTOR

Description of the task:	**Theme:** Arithmetic & Algebra
Simplifying algebraic expressions to find a common factor and evaluating the resulting monomial.	**Concepts and processes** • manipulating numerical and algebraic expressions • performing context-related calculations with integral exponents (rational base) and fractional exponents • finding the common factor

A. Example of an appropriate solution

The following expressions are simplified as follows:

$16x^3y^4 + 3x^3y^4 - 4x^3y^4$	$15x^3y^4$
$\sqrt{25x^8y^6}$	$5x^4y^3$
$(4x^{-3}y^2)(5x^{10}y\ \ )$	$20x^7y^3$
$\dfrac{20x^{-3}y^4}{4x^{-8}y^2}$	$5x^5y^2$

The greatest common factor that can be factored from these four resulting monomials is: $5x^2y^3$

Given that x= 4.

Given that y = 3.

The numerical value of the greatest common factor is:

$$5x^3y^2 = 5(4)^3(3)^2$$
$$= 2880$$

3. MYSTIC AQUARIUM

Description of the task: Finding the dimensions of an equivalent solid	**Theme:** Geometry & Arithmetic **Concepts and processes** • Equivalent solids ○ cube root

A. Example of an appropriate solution

Old aquarium
Volume of the rectangular-based right prism
$V = l \times w \times h$
$= 2 \times 1.4 \times 1.5$
$= 4.2\ \text{m}^3$

New aquarium
$h = 2.6r$

Volume of the circular right cylinder
$V = \pi r^2 h$

Since the capacities of these two aquariums are equivalent, the volume of the circular right cylinder will also be $4.2\ \text{m}^3$:

$4.2 = \pi r^2(2.6r)$
$4.2 = 8.168\ r^3$
$0.51 = r^3$
$0.8 = r$

The radius of the new aquarium is 0.8 m, which means that the diameter is 1.6 m and the height is 2.08 m.

4. LAWN CARE

Description of the task:	Theme: Geometry & Algebra
Find the area of grass to calculate the cost of fertilizer.	**Concepts and processes** • Multiplying/dividing algebraic expression • Pythagorean theorem • Area of decomposable solid

A. Example of an appropriate solution

Step 1: Height of trapezoid

Find the base of the right-angled triangle:
20.4 m – 9.6 m = 10.8 m

Apply Pythagorean Theorem to find height of the trapezoid.

$$a^2 + b^2 = c^2$$
$$(10.8)^2 = h^2 = 182$$
$$116.64 = h^2 = 324$$
$$h^2 = 324 - 116.64$$
$$h^2 = 207.36$$
$$h = 14.4 \text{ m}$$

Step 2: Area of trapezoid

$$A = \frac{(B + b) \times h}{2}$$
$$A = \frac{(20.4 + 9.6) \times 14.4}{2}$$
$$A = 216 \text{ m}^2$$

Step 3 : Solving for the value of *x* (use information from BBQ):

Apply Pythagorean Theorem to find the value of *x*.

$$a^2 + b^2 = c^2$$
$$(0.54x)^2 + (3.6)^2 = (0.9x)^2$$
$$0.2916x^2 + 12.96 = 0.81x^2$$
$$0.5184x^2 = 12.96$$
$$x^2 = 25$$
$$x = 5$$

Step 4: Area of BBQ (Area of triangle)

$$A = \frac{b \times h}{2}$$
$$A = \frac{(0.54 \times 5) \times 3.6}{2}$$
$$A = \frac{9.72}{2}$$
$$A = 4.86 \text{ m}^2$$

Step 5: Area of Flower Bed (Area of rectangle)

$$A = \ell \times w$$
$$A = (2(5) - 6)(5 - 2)$$
$$A = (4)(3)$$
$$A = 12 \text{ m}^2$$

Step 6: Area of grass

Area of trapezoid – Area BBQ – Area Flower Bed
216 – 12 – 4.86 = 199.14 m^2

Step 7: Cost to fertilize grass

$$199.14 \text{ m}^2 \times \frac{\$0.80}{\text{m}^2} = \$159.31$$

Note: Accept $159.32 because some students will argue that businesses always round up.

5. FUNDRAISING

Description of the task:
Determine number of bicycles to sell in order to make a greater profit than the girls.
	Theme: Algebra
	
	Concepts and processes
	• Polynomial function degree 1
	  ○ partial variation
	  ○ direct variation
	• System of first degree equation

A. Example of an appropriate solution

Step 1: Determine the amount of money raised by Group A

Let x_1 be the number of pieces of jewellery sold and $y1$ be the net profit from the jewellery sales

We have points given: (85,480) and (120,760)

Use the two data points to calculate the rate of change: $a = \frac{\Delta y}{\Delta x} = \frac{760-480}{120-85} = \frac{280}{35} = 8$

Since each piece of jewellery is sold at a fixed price the relation can be expressed as a linear function in the for $y = ax + b$

Using the slope and one other point, solve for b.

$$y = ax + b$$
$$760 = (8)(120) + b$$
$$760 = 960 + b$$
$$760 - 960 = b$$
$$-200 = b$$

Therefore the rule of this relation is: $y = 8x - 200$

Use the function rule to determine money raised when 90 pieces of jewellery are sold:

$$y = 8x - 200$$
$$y = (8)(90) - 200$$
$$y = 720 - 200$$
$$y = \$520$$

Step 2: Minimum number of bicycles sold

Let x_2 be the number of bicycles sold and y_2 be the net profit from bicycle sales

From the graph we have two points: (0,0) and (4,104)

Use the two data points to calculate the rate of change: $a = \frac{\Delta y}{\Delta x} = \frac{104 - 0}{4 - 0} = \frac{104}{4} = 26$

The graph illustrates a direct linear function therefore the rule can be expressed as $y = ax$

Therefore the rule of this relation is: $y = 26x$

Use the function rule to determine the number of bicycles sold if the money raised exceeds $520:

$520 = 26x$

$\frac{520}{26} = x$

$20 = x$

Group B must sell more than 20 bicycles.

OR Group B must sell at least 21 bicycles.

6. MAKING THE POOL SAFE

Description of the task: Determining the cost of gating a pool	**Theme:** Geometry & Algebra **Concepts and processes** • Area of figures that can be split into sectors • Direct variation • Multiplying/dividing polynomials

A. Example of an appropriate solution

Algebraic area of the pool

$(2x + 4)(x + 1) = 2x^2 + 6x + 4$

Dimension of exterior rectangle

L: $2x + 4 + 4 = (2x + 8)$
W: $x + 1 + 3 = (x + 4)$

Total area with gravel path

$(2x + 8)(x + 4) = 2x^2 + 16x + 32$

Area of gravel

$(2x^2 + 16x + 32) – (2x^2 + 6x + 4)$
$= 10x + 28$

Value of *x*

$10x + 28 = 48$
$x = 2$

Perimeter of the pool

$2(2x + 4) + 2(x + 1)$
$= 6x + 10$
$= 6(2) + 10$
$= 22$ m

Cost of the fence

22 × 45 = $990

7. NEW YORK CITY

Description of the task: Comparing two linear functions	**Theme:** Arithmetic & Algebra **Concepts and processes** • comparing situations

A. Example of an appropriate solution

Step 1: Find the rule for option 1

Let *x* be the number of nights in NYC
Let *y* be the total cost of the trip

Using two points from the table of values, (6, 750) and (4, 500), calculate the slope:

$$\begin{aligned} a &= \frac{\Delta y}{\Delta x} \\ &= \frac{750-500}{6-4} \\ &= \frac{250}{2} \\ &= 125 \end{aligned}$$

Use the slope and one point to solve for b

$$\begin{aligned} y &= ax+b \\ y &= 125x+b \\ 750 &= 125(6)+b \\ 750-750 &= b \\ b &= 0 \end{aligned}$$

Add \$250 for transportation and option 1 can be represented by the equation:
$y = 125x + 250$

Step 2: Find the rule for option 2

The transportation and membership fees are the fixed costs(b).

The $150 per night is the rate therefore option 2 can be represented by the equation $y = 150x + 175$

Number of nights	Total cost for option 1 ($)	Total cost for option 2 ($)
1	375	325
2	500	475
3	625	625
4	750	775
5	875	925

If Luka stays in NYC for 1 or 2 nights, option 2 is cheaper than option 1.

If Luka stays in NYC for 3 nights, both options are equal since the total cost is the same.

If Luka stays in NYC for 4 or more nights, option 1 is cheaper than option 2.

8. CHARITY REPORT

Description of the task:	Theme: Statistics and Probability
Correcting statements about charity donations for a written report.	**Concepts and processes** • Graphs: box-and-whisker Plot • Measures of central tendency: mode, • median and weighted mean • Measures of dispersion: range of each • part of a box-and-whisker plot (including • interquartile range) ○ Calculating measures of central tendency and of dispersion • Comparing distributions

A. Example of an appropriate solution

The following can be deduced from the information provided:

2008:

- Mean was $318.29.
- The same number of people donated between $300 and $5000 as donated between $5 and $75.
- The maximum donation was in the range of [2750, 3000[, but the exact maximum value is unknown.

2009:

- We do not have enough information from just the box and whisker chart to determine the mean.
- The $100 indicates the median donation.
- The box and whisker plot is divided in quartiles, therefore the same number of people donated between $5 and $75 as the number of people that donated between $300 and $5000.
- The maximum donation was $5000.

The following are examples of correct statements:

1. **The mean donation from 2008 is $318.29, not $300. We do not have enough information from the chart given to find the mean donation from 2009, but we know that $100 is the median, and not the mean.**

2. **The same number of people made donations between $300 and $5000 as made donations from $5 to $75.**

3. **In 2008, the maximum donation was in the range [2750, 3000[. In 2009, the maximum donation was $5000.**

9. <u>THE AREA OF REGULAR POLYGONS</u>

Description of the task:	**Theme:** Geometry
Make a conjecture comparing the number of sides of a regular polygon and its area	**Concepts and processes** • Pythagorean theorem • Area of regular polygons • Perimeter of regular polygons

A. Example of an appropriate solution

If the perimeter is set to 24 units then:

For the triangle:

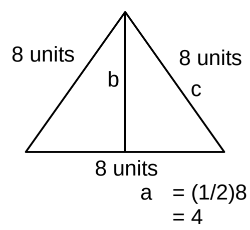
Let *x* be the length of the sides:

$P = 3x$

$24 = 3x$

$\frac{24}{3} = x$

$8 = x$



$c^2 = a^2 + b^2$

$b^2 = 8^2 - 4^2$

$b^2 = 64 - 16$

$b^2 = \sqrt{48}$

$b \approx 6.93$ units

$A = \frac{base \times height}{2}$

$A = \frac{8 \times 6.93}{2}$

$A \approx 27.71 \text{ units}^2$

For the square or diamond:

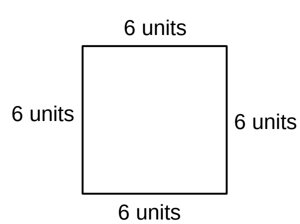
Let *x* be the length of the sides:

$P = 4x$

$24 = 4x$

$\frac{24}{4} = x$

$6 = x$



$A = base \times height$

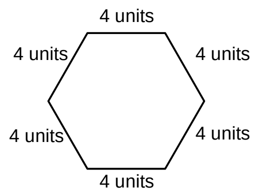
$A = 6 \times 6$

$A = 36 \text{ units}^2$

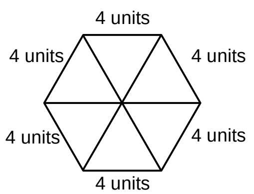
For the hexagon:

Let *x* be the length of the sides:

$$P = 6x$$
$$24 = 6x$$
$$\frac{24}{6} = x$$
$$4 = x$$

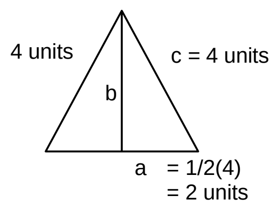


A hexagon is made up of six (6) equilateral triangles:



There the apothem can be calculated using the Pythagorean Theorem:

$$c^2 = a^2 + b^2$$
$$b^2 = 4^2 - 2^2$$
$$b^2 = 16 - 4$$
$$b^2 = \sqrt{12}$$
$$b \approx 3.46 \text{ units}$$



$$A_{\text{triangle}} = \frac{base \times height}{2}$$
$$A_{\text{triangle}} = \frac{4 \times 3.46}{2}$$
$$A_{\text{triangle}} \approx 6.92 \text{ units}^2$$

$$A_{\text{Hex}} = A_{\text{triangle}} \times 6$$
$$A_{\text{Hex}} \approx 6.92 \times 6$$
$$A_{\text{Hex}} \approx 41.52$$

Summary:

Shape	**Perimeter**	**Number of sides**	**Area**
Triangle	24 units	3	27.71 units^2
Square	24 units	4	36 units^2
Hexagon	24 units	6	41.52 units^2

Conjecture: For regular polygons of equal perimeter, as you increase the number of sides the area will also increase.