

MATHEMATICS

563-306

**PROTOTYPE
EXAMINATION
2008**

Mathematics Prototype Examination Secondary Cycle Two

Part II

Name: _____

Class Number: _____

School: _____

Time: 3 hours

Éducation,
Loisir et Sport

Québec 

The following are the criteria that will be used to evaluate your level of competency development in the different situations presented in this booklet.

<i>USES MATHEMATICAL REASONING</i> EVALUATION CRITERIA
Cr. 3 - Proper implementation of mathematical reasoning suited to the situation
Cr. 2 - Correct application of concepts and processes suited to the situation
Cr. 4 - Proper organization of the steps in a proof suited to the situation
Cr. 5 - Correct justification of the steps in a proof suited to the situation
Cr. 1 - Formulation of a conjecture appropriate to the situation

<i>COMMUNICATES BY USING MATHEMATICAL LANGUAGE</i> EVALUATION CRITERIA
Cr. 2 - Correct interpretation of a mathematical message involving at least two registers of semiotic representation
Cr. 1 - Correct translation of a mathematical concept or process into another register of semiotic representation
Cr. 3 - Production of a message appropriate to the communication context
Cr. 4 - Production of a message in keeping with the terminology, rules and conventions of mathematics

Instructions

1. Fill in all the required information in the spaces provided in this booklet.
2. There are 12 questions in this booklet. For each question, you must demonstrate your reasoning to justify your answer. The steps in your procedure must be organized and clearly presented.
3. You are permitted to use graph paper, a ruler, a compass, a set square, a protractor and a calculator with or without a graphic display.
4. You may refer to the memory aid you prepared on your own before the examination. The memory aid consists of one letter-sized sheet of paper ($8\frac{1}{2} \times 11$). Both sides of the sheet may be used. Any mechanical reproduction of this memory aid is forbidden. All other reference materials are forbidden.

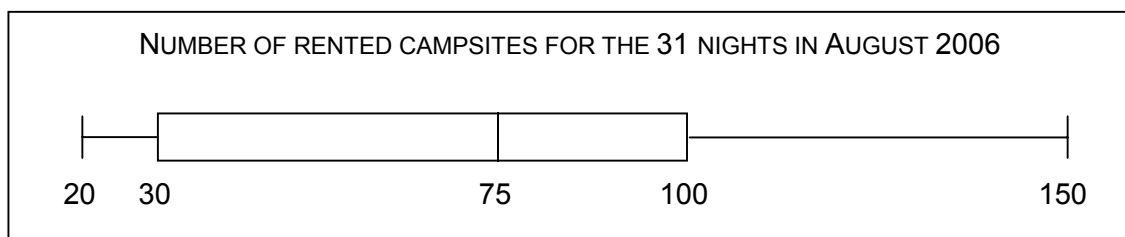
Note: Figures are not necessarily drawn to scale.

1. RENTED CAMPSITES

Louise owns a campground with 150 campsites. In 2006, some of these campsites remained unoccupied for several nights. In order to attract campers and increase the number of campsites rented, she decided to organize various fun activities for the following year. In July 2007, Louise set up a small water park for campers in her campground.

At the end of the 2007 camping season, Louise compared her income from both summers, and saw an increase in revenue. She would like to know if this is related to an increase in campsite rentals.

The campsite rental data she has from August 2006 has been graphed in a box-and-whisker plot, shown below.



Each day in August 2007, Louise noted the number of rented campsites on a calendar page, as illustrated below.

August 2007						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Rented Campsites			1 135	2 145	3 150	4 150
5 110	6 115	7 118	8 120	9 120	10 130	11 150
12 95	13 85	14 75	15 65	16 50	17 132	18 150
19 90	20 45	21 45	22 42	23 40	24 140	25 150
26 40	27 35	28 35	29 35	30 20	31 130	

Louise wants you to analyze the data and to tell her whether the number of rented campsites increased after the water park opened. She also wants you to explain your conclusions.

Show all your work

Conclusions and Explanations

***Communicates by using
mathematical language***

**Evaluation
criteria**

***Observable
indicators
corresponding
to level...***

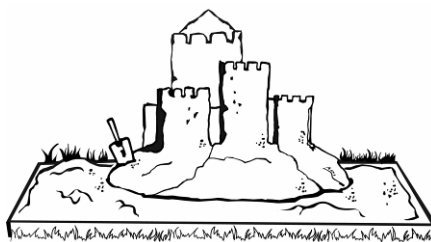
Cr. 2

Cr. 1

Cr. 3

Cr. 4

2. SAND FOR EVERYONE



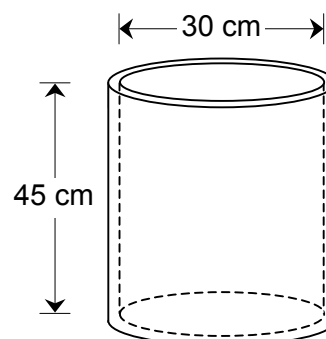
A sandcastle building contest is being held at the campground this morning.

Three types of teams have signed up for the contest. The table below shows the types of teams and the number of teams of each type that signed up for the contest.

TYPE OF TEAM	NUMBER OF TEAM MEMBERS	NUMBER OF TEAMS SIGNED UP FOR THE CONTEST
A	2	20
B	3	15
C	4 or 5	40

Sand was given to teams in congruent cylindrical containers. The inner diameter of each container is 30 cm. The inner height is 45 cm.

All the containers given to the teams are completely filled with sand. The total amount of sand in all the containers is 18 m^3 .



The number of containers given to each of the 75 teams depended on the type of team.

- ◆ Each type B team received 1 more container than each type A team.
- ◆ Each type C team received twice as many containers as each type A team.

Now that the containers have been distributed, a certain amount of sand remains. Without taking part in the contest, some campground employees want to use this sand to build their own sand castle.

How many buckets of sand do the campground employees have for building their castle?

Show all your work

The campground employees have _____ buckets of sand to build their castle.

Uses mathematical reasoning	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

3. CAMPSITE CLOSINGS

A certain region has 7 campgrounds. We are interested in the number of campsites each campground has. The smallest has only 68 campsites, while the largest has 200 campsites.

Each campground closed 8 campsites in order to set up games.

A journalist made the following claim:

Following the recent campsite closings, the mean number of campsites in these campgrounds decreased by 8, whereas the median number of campsites remained the same.

Is the journalist right or wrong?

Explain your answer.

Show all your work

The journalist is right
because _____ wrong ,

_____.

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

4. CARDBOARD OR PAPER

Mr. Gladstone owns a campground, and he wants to print advertising fliers to be distributed at various travel and leisure shows. The local printer suggests that he choose one of two types of fliers: a sheet of cardboard printed front and back or a sheet of paper printed front and back and folded in three. Mr. Gladstone wants to pay as little as possible for the fliers.

The table of values below gives examples of the cost according to the type and number of fliers printed. The cost of the fliers is dependent on the number of fliers printed.

Number of fliers printed	Cost	
	Cardboard fliers	Folded paper fliers
3 750	\$275	\$417
5 000	\$300	\$432
9 250	\$385	\$483
12 250	\$445	\$519

Mr. Gladstone hasn't decided yet how many fliers he will need, but he does not plan to purchase more than 25 000.



Show Mr. Gladstone how to decide which company to purchase fliers from, depending on how many fliers he will purchase.

Show all your work

Mr. Gladstone should choose _____

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

5. THE TARGET

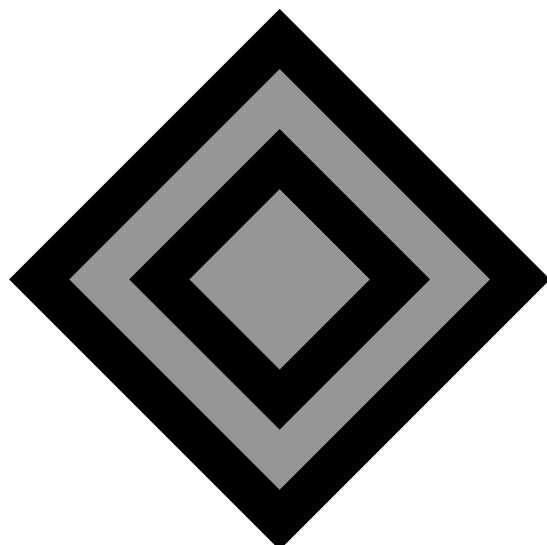
On rainy days, campers can get together and have fun in the main recreation room, where the owner of the campground wants to set up a new dartboard.

- ♦ The target is square.
- ♦ The square at the centre of the target is grey and measures 3 cm by 3 cm.
- ♦ Around this square are bands 1 cm wide in alternating colours: a black band, then a grey band, then another black band, etc.

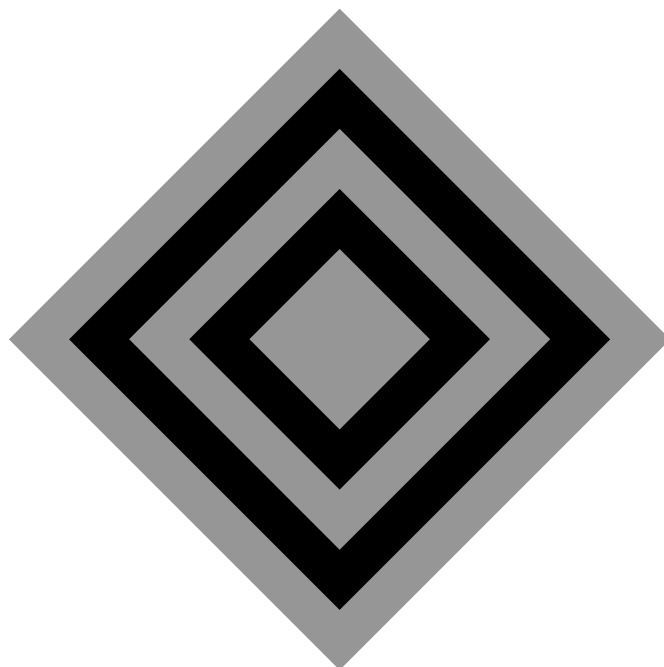
A randomly thrown dart hits the target.

Make a conjecture about which colour has the greatest probability of being hit by the dart depending on how many bands there are around the square at the centre of the target.

TARGET WITH 3 BANDS



TARGET WITH 4 BANDS



Show all your work

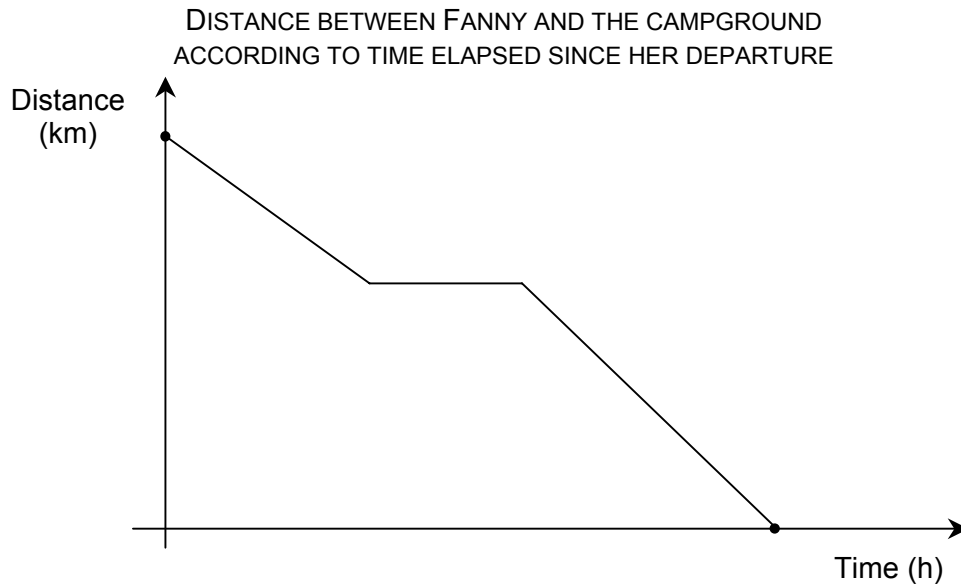
Conjecture

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

6. A MOTORCYCLE RIDE

Fanny and Gabriel live in different towns. They agreed to meet at a campground where they plan to spend the weekend. They left their respective homes at the same time.

Fanny lives 160 km from the campground and made the trip by motorcycle. During the first hour, she travelled at a constant speed of 60 km/h. After the first hour, she stopped to rest for 45 minutes. She then continued on to the campground, travelling at a constant speed of 80 km/h. The graph below gives a general idea of the distance between Fanny and the campground.



The distance between Gabriel and the campground can be represented by function g , where

x : time elapsed, in hours, since Gabriel's departure

$g(x)$: distance, in kilometres, between Gabriel and the campground

Below are three facts about Gabriel's motorcycle ride and function g .

- ◆ Gabriel's rate of speed did not change during his trip.
- ◆ Gabriel's house is 40 km closer to the campground than Fanny's house.
- ◆ Gabriel arrived at the campground 1 hour before Fanny did.

Accurately graph function g .

Show all your work

A large grid of graph paper for showing work, consisting of a 30x30 grid of small squares.

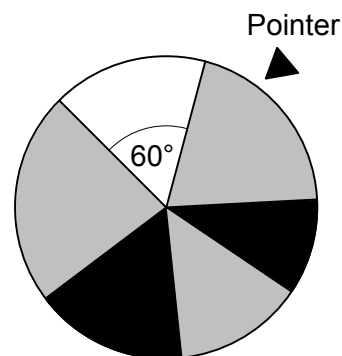
<i>Communicates by using mathematical language</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 2	
Cr. 1	
Cr. 3	
Cr. 4	

7. THE WHEEL

A campground owner offers each camper renting a campsite for one night the opportunity to play a game of chance called *THE WHEEL*. Winning contestants get a 25% discount on their campsite rental fee the next time they come to this campground for one night.

DESCRIPTION OF THE WHEEL

You spin the wheel of fortune shown on the right. When the wheel stops spinning, the probability that the pointer will be pointing at a black sector is $\frac{4}{15}$. The central angle of the white sector measures 60° .



THE GAME

To begin with, contestants must decide whether they will spin the wheel 1, 2 or 3 times.

The conditions for winning depend on the number of times contestants choose to spin the wheel.

For contestants to win, the pointer must be pointing at:

- ♦ the white sector if they have chosen to spin the wheel only once
- ♦ a black sector after each turn if they have chosen to spin the wheel twice
- ♦ a grey sector after each turn if they have chosen to spin the wheel three times

Your family rents a campsite in this campground for one night. Your father asks you for advice on how to maximize his probability of winning.

What would you advise your father to do?

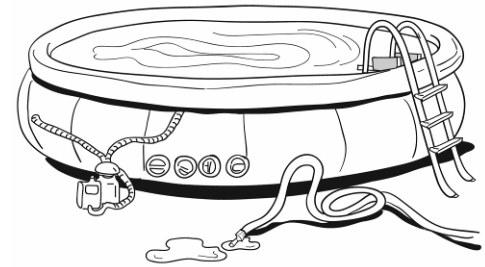
Show all your work

Your advice

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

8. JULIA'S SWIMMING POOL

Every spring, Julia sets up an aboveground swimming pool in her backyard and fills it with water. The amount of time it takes to fill the pool depends on the total flow rate of the combination of hoses used.



The table of values below shows the filling time associated with the total flow rate of the hoses used in the 5 years following the installation of the swimming pool.

Total flow rate of the hoses (litres/hour)	Filling time (h)
480	32.5
600	26
960	16.25
1040	15
1560	10

This year, Julia filled her swimming pool in 24 hours.

What was the total flow rate of the hoses Julia used to fill her swimming pool this year?

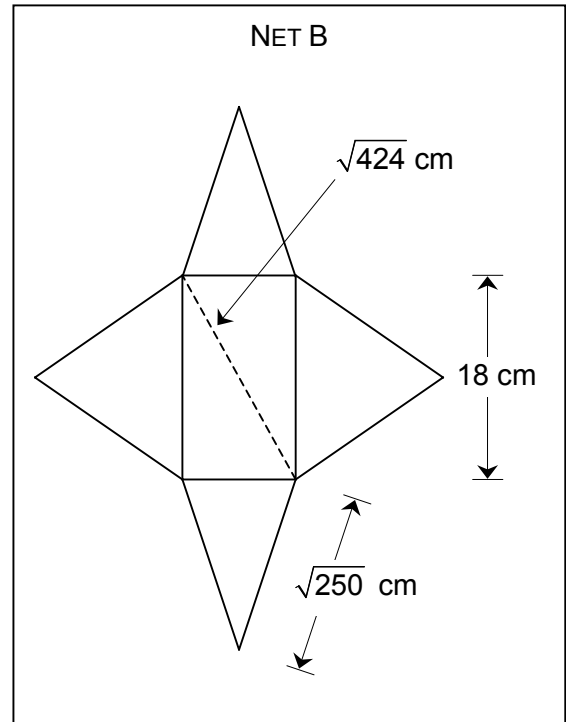
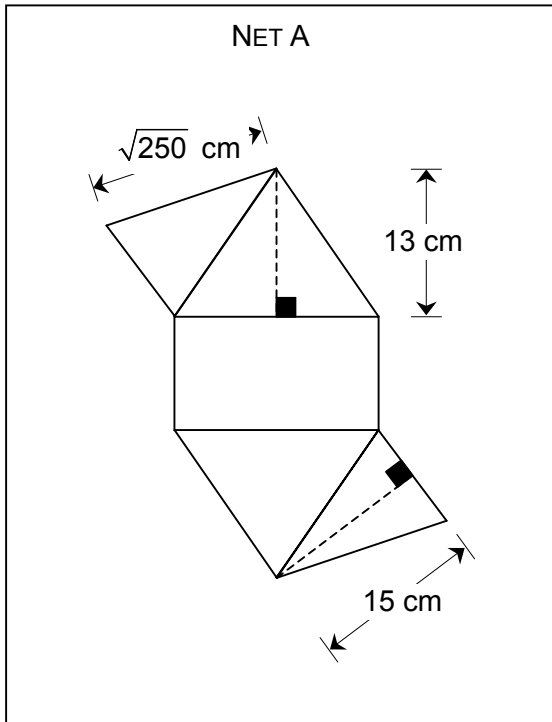
Show all your work

The total flow rate of the hoses Julia used to fill her swimming pool
this year was _____ litres per hour.

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

9. PYRAMID NETS

Shown below are the nets of two right pyramids, each with a rectangular base.



Emily claims that these two pyramids are congruent.

Is she right?

Explain your answer.

Show all your work

Emily is right

Emily is wrong ,

because _____

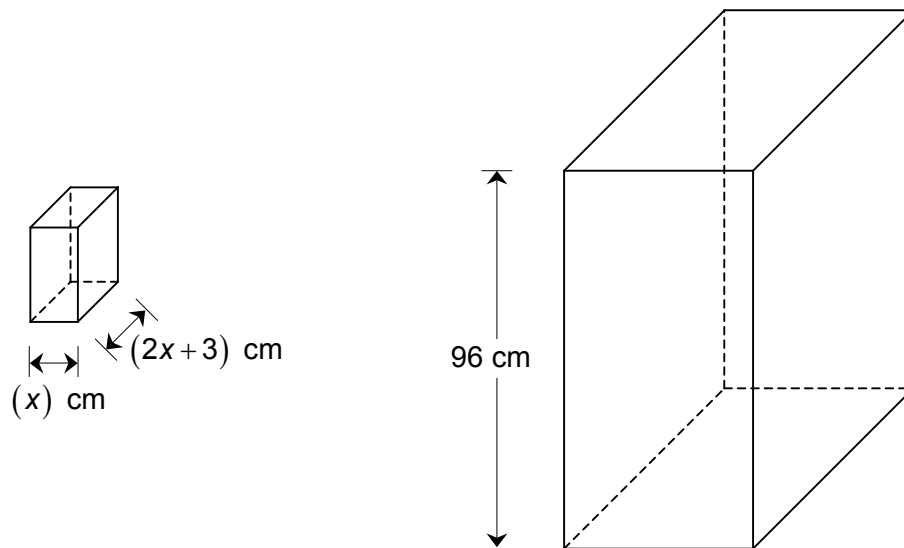
<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

10. SIMILAR PRISMS

Shown below are two similar right prisms, each with a rectangular base.

The lengths, in centimetres, of the sides of the base of the smaller prism can be represented by the monomial (x) and the binomial $(2x + 3)$ respectively. In this case, the volume of the smaller prism, in cm^3 , is represented by the polynomial $(6x^3 + 9x^2)$.

The height of the larger prism is 96 cm. The total area of the larger prism is 64 times the area of the smaller prism.



What is the numerical volume of the larger prism?

Show all your work

The numerical volume of the larger prism is _____ .

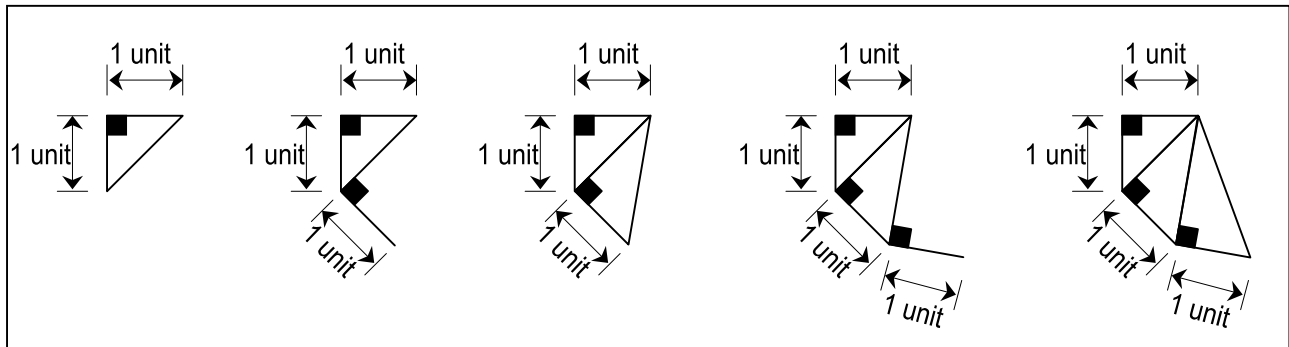
Uses mathematical reasoning	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

11. THE PYTHAGOREAN SPIRAL

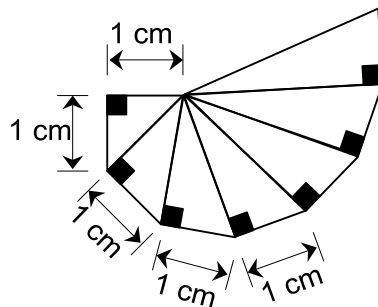
A Pythagorean spiral is a figure composed of a series of right triangles. Every two consecutive triangles have a common side. All the triangles in a Pythagorean spiral have a common vertex. In each triangle, the side opposite this vertex measures 1 unit.

The procedure for constructing a Pythagorean spiral is as follows:

- Step 1 Draw a right triangle in which the legs of the right angle each measure one unit.
- Step 2 Draw a line segment one unit long and perpendicular to the hypotenuse of the triangle you have just drawn. This segment and the side measuring one unit in the preceding triangle have a common endpoint.
- Step 3 Draw a hypotenuse to form a new right triangle.
- Step 4 Repeat steps 2 and 3.



Here are the first 7 triangles in a Pythagorean spiral measured in centimetres.



The length of the hypotenuse of some of the triangles in a Pythagorean spiral is a whole number.

We want to determine the number of triangles located between two triangles that have hypotenuses whose lengths are consecutive whole numbers.

If n represents a natural number, what algebraic expression represents the number of triangles in the Pythagorean spiral that are located between the triangle whose hypotenuse measures (n) cm and the triangle whose hypotenuse measures $(n + 1)$ cm?

Show all your work

If n represents a natural number, then the expression _____ represents the number of triangles in the Pythagorean spiral that are located between the triangle whose hypotenuse measures (n) cm and the triangle whose hypotenuse measures $(n + 1)$ cm.

Uses mathematical reasoning	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	

12. SERIES OF EXPRESSIONS

The table below shows a series of algebraic expressions.

1st expression	$\frac{2^6}{2^5}x$
2nd expression	$(2x - 6)(x + 3) + 18$
3rd expression	$\frac{100^{\frac{1}{2}}x^8}{5x^5}$
4th expression	$\frac{x^4}{2^{-1}}$
5th expression	$(x^5 + 3x^3 - 5) - (4x^3 - 5) + (x^5 + x^3)$
6th expression	$\frac{10x^{16}}{?}$

What is a possible denominator for the 6th expression?

Show all your work

A possible denominator for the 6th expression is _____.

<i>Uses mathematical reasoning</i>	
Evaluation criteria	Observable indicators corresponding to level...
Cr. 3	
Cr. 2	
Cr. 4	
Cr. 5	
Cr. 1	