

Philemon Wright High School

Mathematics – Grade 9

Situational Problem 563-306

January 2011

The Cabinet Maker

Name: ANSWER KEY

Group: 306-_____

Time: 2 hours

| Solves a situational problem | | | | | | |
|--|---|----|----|----|----|---|
| Observable indicators corresponding to level | | 5 | 4 | 3 | 2 | 1 |
| Evaluation Criteria | Cr.1 – Shows understanding of the problem | 40 | 32 | 24 | 16 | 8 |
| | Cr.2 – Uses knowledge that is applicable to the problem | 40 | 32 | 24 | 16 | 8 |
| | Cr.3 – Procedure and answer | 20 | 16 | 12 | 8 | 4 |
| | Cr.4 – Validation of the steps | | | | | |

General instructions

1. Fill in all the required information in the spaces provided in this booklet.
2. You are permitted to use your geometry set, your calculator, and your memory aid.
3. All other reference materials are forbidden.
4. If you get stuck on one part, continue working on the rest of the problem. You are trying to show that you know what to do, even if some of your numbers are wrong.
5. You have 2 hours to complete your work.

Evaluation criteria

1. Cr.1 – Oral or written indication that the student has an appropriate understanding of the situational problem.
2. Cr.2 – Mobilization of mathematical knowledge appropriate to the situational problem.
3. Cr.3 – Development of a solution (includes procedure and final answer) appropriate to the situational problem.
4. Cr.4 – Appropriate validation of the steps in the solution.

The Cabinet Maker

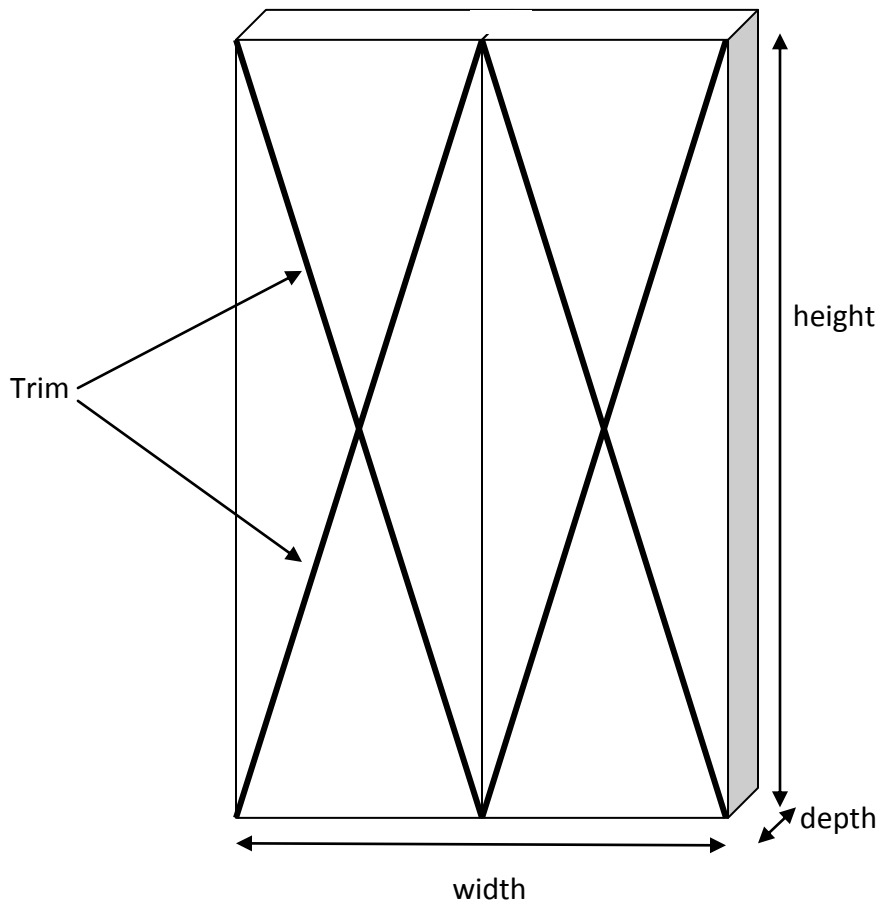
You have been hired by a cabinet maker to calculate the exact cost of producing a new series of armoires he intends to build. You have been assigned a budget of \$4 500 (all costs given include the different taxes), to complete the following tasks based on cost:

- Figure out the formula to quickly calculate the quantity of wood needed to build an armoire.
- Choose the type and calculate the cost of wood used for the armoires.
- Calculate the cost of the trim used to decorate the armoires.
- Choose how many coats of varnish can be used.
- Calculate the exact cost of purchasing all the materials to complete the project.

You must then submit a detailed report which will be reviewed by the cabinet maker. The report must include the materials you've selected and the total cost of production of the armoires.

The cabinet maker wants to build 12 armoires. The armoires are shaped like rectangular-based prisms. To estimate the costs, you have to take into consideration that wood is needed on all 6 sides of the armoires (prisms).

Armoire



(Figure not to scale)

The trim corresponds to the two X-shapes on the diagram above.

The dimensions used by the cabinet maker to build the armoires are always the same: the width is 3 times greater than the depth plus 4, and the height is 7 times greater than the depth minus 20, when the dimensions are in centimetres.

The formula

The cabinet maker wants you to figure out the formula that would give the total area of a cabinet. This way, he could use an Excel-type spreadsheet to quickly calculate how much wood is needed to build one armoire.

(Reminder: Area of a rectangle = length x width)

Work

$$\text{Width} = 3x + 4$$

$$\text{Depth} = x$$

$$\text{Height} = 7x - 20$$

Total area for 1 armoire:

$$\text{Front and Back: } 2 \times \text{width} \times \text{height} = 2(3x + 4)(7x - 20) = 42x^2 - 64x - 160$$

$$\text{Left and Right: } 2 \times \text{depth} \times \text{height} = 2(x)(7x - 20) = 14x^2 - 40x$$

$$\text{Top and Bottom: } 2 \times \text{width} \times \text{depth} = 2(3x + 4)(x) = 6x^2 + 8x$$

$$\text{Total area} = (42x^2 - 64x - 160) + (14x^2 - 40x) + (6x^2 + 8x) = 62x^2 - 96x - 160$$

So the formula is: $62x^2 - 96x - 160$

Wood costs

To help you figure out how much wood is needed, the cabinet maker informs you that for this armoire, the sum of the 3 dimensions comes up to exactly 314 centimetres.

The cabinet maker also tells you that he does not want to spend less than 50% of his budget, or more than 75%, on wood.

Here are the wood types you can choose from:

Wood types

| <u>Type</u> | <u>Price (\$/m²)</u> |
|---------------------|---------------------------------|
| Teak | 18.00 |
| Rosewood | 25.00 |
| Brazilian Tulipwood | 50.00 |
| Ebony | 70.00 |

(Reminder: 1 m = 100 cm and 1 m² = 10 000 cm², Area of a rectangle = length x width)

Work

$$\text{Width} = 3x + 4$$

$$\text{Depth} = x$$

$$\text{Height} = 7x - 20$$

$$3x + 4 + x + 7x - 20 = 314$$

$$11x - 16 = 314$$

$$11x = 330$$

$$x = 30 \text{ cm}$$

Costs:

$$\text{Teak: } 63.312 \times 18.00 = \$1139.62$$

$$\text{Brazilian Tulipwood: } 63.312 \times 50.00 = \$3165.60$$

$$50\% \text{ of budget} = 0.50 \times 4500 = \$2250$$

Only the Brazilian Tulipwood falls in that range.

Wood cost: \$3165.60

$$\text{Width} = 3(30)+4 = 94 \text{ cm or } 0.94 \text{ m}$$

$$\text{Depth} = 30 \text{ cm or } 0.30 \text{ m}$$

$$\text{Height} = 190 \text{ cm or } 1.90 \text{ m}$$

Total area for 1 armoire:

$$\text{Front and Back: } 0.94 \times 1.90 \times 2 = 3.572 \text{ m}^2$$

$$\text{Left and Right: } 0.30 \times 1.90 \times 2 = 1.14 \text{ m}^2$$

$$\text{Top and Bottom: } 0.94 \times 0.30 \times 2 = 0.564 \text{ m}^2$$

$$\text{Total} = 5.276 \text{ m}^2$$

$$12 \text{ armoires: } 12 \times 5.276 = 63.312 \text{ m}^2$$

$$\text{Rosewood: } 63.312 \times 25.00 = \$1582.80$$

$$\text{Ebony: } 63.312 \times 70.00 = \$4431.84$$

$$75\% \text{ of budget} = 0.75 \times 4500 = \$3375$$

The trim and the hardware

The trim is sold in strips of 4 metres at a cost of \$28.00 per strip. For each armoire, the cabinet maker will need 24 screws at \$0.15 each, 2 handles at \$6.50 each, and 4 hinges at \$2.75 each. You need to figure out the total cost for the trim and the hardware.

Work

Hardware for 1 armoire:

$$24 \times 0.15 = \$3.60$$

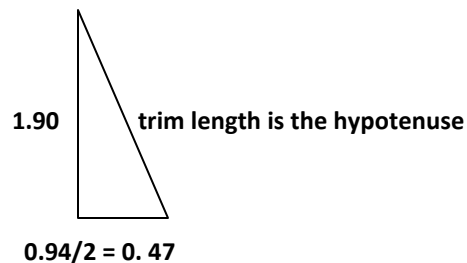
$$2 \times 6.50 = \$13.00$$

$$4 \times 2.75 = \$11.00$$

$$\text{Total} = \$27.60$$

$$\text{For 12 armoires} = 12 \times 27.60 = \underline{\$331.20}$$

Trim:



$$C^2 = a^2 + b^2$$

$$C^2 = 1.90^2 + 0.47^2$$

$$C^2 = 3.8309$$

$$C \approx 1.957 \text{ m}$$

4 lengths needed x 12 armoires:

$$4 \times 12 \times 1.957 \approx 93.95 \text{ m}$$

Number of strips needed: $93.95 / 4 \approx 23.49$ so 24

$$\text{Cost of the strips: } 24 \times 28.00 = \underline{\$672.00}$$

| | | | | | | |
|---------------|-----------|---|----------|---|----------|-------------|
| Costs so far: | Wood | + | Trim | + | Hardware | |
| | \$3165.60 | + | \$672.00 | + | \$331.20 | = \$4168.80 |

The varnish

The varnish costs \$12.00 per litre and each litre of varnish covers 8 m². You need to figure out how many coats of varnish can be used to stay within budget.

Work

$$\text{Money left} = \$4500 - 4168.80 = \$331.20$$

$$\text{Total area for 12 armoires} = 63.312 \text{ m}^2$$

$$\text{Number of litres needed} = 63.312 / 8 = 7.914 \text{ so we round up to 8 litres.}$$

$$\text{Cost of varnish for 1 coat (12 armoires)} = 8 \text{ litres} * 12.00 \text{ \$/L} = \$96.00$$

Let x represent the number of coats that can be applied,

$$96x \leq 331.20$$

$$\underline{96x} \leq \underline{331.20}$$

$$96 \quad 96$$

$$x \leq 3.45$$

So only 3 coats of varnish can be applied.

The final report

Write your final report to the cabinet maker on this page.

The formula

Wood costs

The trim and the hardware

The varnish
